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International Journal of Heat and Mass Transfer 48 (2005) 3231–3243
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Transient response of plate heat exchangers considering effect of flow maldistribution

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Received 24 September 2004

Abstract

Plate heat exchangers have been playing important role in the power and process industries in the recent past. Hence, it is important to develop simulation strategies for plate heat exchangers accurately. This analysis represents the dynamic behaviour of the single pass plate heat exchangers, considering flow maldistribution from port to channel. In addition to maldistribution the fluid axial dispersion is used to characterise the back mixing and other deviations from plug flow. Due to unequal distribution of the fluid, the velocity of the fluid varies from channel to channel and hence the heat transfer coefficient variation is also taken into consideration. Solutions to the governing equations have been obtained using the method of Laplace transform followed by numerical inversion from frequency domain. The results are presented on the effects of flow maldistribution and conventional heat exchanger parameters on the temperature transients of both U-type and Z-type configurations. It is found that the effect of flow maldistribution is significant and it deteriorates the thermal performance as well as the characteristic features of the dynamic response of the heat exchanger. In contrast to the previous studies, here the axial dispersion describes the inchannel back mixing alone, not maldistribution, which is physically more appropriate. Present method is an efficient and consistent way of describing maldistribution and back mixing effects on the transient response of plate heat exchangers using an analytical method without performing intensive computation by complete numerical simulation.

Keywords: Plate heat exchanger; Maldistribution; Dispersion; Backmixing

1. Introduction

Plate heat exchangers (PHE) have been the most successful type of heat exchangers, which have taken major strides in the last three decades. Though this type of heat exchangers were specially designed for hygienic applications such as dairy, brewery and food processing industries, it has also found place in the modern power and process industries due to its multi-fold advantages such as compactness, flexibility, ease of installation, capability to recover heat with extremely small temperature difference, and smaller hold up volume (hence quicker response to control operations). Due to its expanding

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0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2005.02.032

Nomenclature

A	heat transfer area per effective plate [m^2]
$\overline{A}$	coefficient matrix for system of differential equations
A_c	free flow area in a channel [m^2]
$\overline{B}$	diagonal matrix, Eq. (41)
C	heat capacity of resident fluid (s) [J K^{-1}]
C_p	specific heat capacity of fluid (s) [J kg^{-1} K^{-1}]
D	axial dispersion coefficient [W m^{-1} K^{-1}]
$\overline{D}$	matrix resulting from boundary conditions
d_j	elements of matrix $\overline{D}$
$f(Z)$	inlet temperature function
$F(s)$	Laplace transform of $f(Z)$
G	transfer function in first order system
h	heat transfer coefficient [W m^{-2} K^{-1}]
i	square root of -1
K	gain in the first order system
K_i	$\frac{\mathrm{NTU}_i}{2}R_{Wi}(R_2.R_N)^{m_{i+1}}$
l_c	path traversed by fluid particle between two consecutive channels [m]
L	fluid flow length in channel [m]
m	maldistribution parameter, Eq. (13)
m_j	$j-2[j/2]$, where j is an integer
$\dot{m}$	mass flow rate in the conduit [kg s^{-1}]
M_i	$R_c \cdot s + K_{i-1} + K_i$
n	number of channels on one side
NTU_1	number of transfer units of the heat exchanger
Pe	axial dispersive Peclet number, $\dot{w}L/A_cD$
R_2	capacity rate ratio in the channels, $\dot{w}_{a2}/\dot{w}_{a1}$
R_{g2}	capacity rate ratio of the combined flow, $\dot{w}_{g2}/\dot{w}_{g1}$
$R_{g\tau}$	characteristic rate ratio of the combined flow, τ_{rg2}/τ_{rg1}
R_N	ratio U_{a2}/U_{a1}
R_c	wall heat capacity rate ratio, C_w/C_1
R_τ	characteristic time ratio in channels, τ_{ra2}/τ_{ra1}
s	transformed time variable in Laplace domain
$\overline{S}$	matrix with inlet fluid function
t	dimensionless temperature, $\frac{\theta-\theta_{g1,in}}{\theta_{g2,in}-\theta_{g1,in}}$
T	non-dimensionless temperature obtained by Laplace transformation of temperature term
$\overline{T}$	temperature matrix, in Eq. (43)
u	a unit step function
$\overline{U}$	matrix of eigen vectors of the matrix A
u_i	elements of matrix U
$U_{a1(2)}$	$(hA/\dot{w}_a)_{1(2)}$
V_i	the velocity of the fluid in the port after ith channel [m s^{-1}]
V_g	velocity of the combined fluid [m s^{-1}]
$\dot{V}_c$	volume flow rate in the channel [m^3 s^{-1}]
$\dot{V}_g$	volume flow rate of the combined fluid [m^3 s^{-1}]
$\dot{v}_c$	non-dimensional channel volume flow rate
$\dot{w}$	thermal capacity rate of the fluid in channel, $\dot{m}C_p$ [W K^{-1}]
X	space coordinate [m]
x	dimensionless space coordinate along the channel, X/L
y	dimensionless space coordinate along the port
z	dimensionless time τ/τ_{ra1}

Greek symbols

β_j	jth eigenvalue of matrix A
θ	temperature
τ	time [s]
τ_{ra}	residence time in case of uniform distribution, $C/\dot{w}_a$
τ_d	delay time
τ_c	time constant
$\Delta\tau$	time of travel in the port
ϕ	dimensionless phase lag (cumulative value)
$\Delta\phi$	dimensionless phase lag (discrete value), $\Delta\tau_i/\tau_{ra1}$

Subscripts

a	the case of uniform flow distribution
g	combined flow before departing into channels
i	ith channel
W	plate
Wi	ith plate
0	initial
1	the fluid in odd channels
2	the fluid in even channels

applications, there is tremendous amount of research activities concerning this particular type of heat exchanger in recent times. This has given rise to an array of new generation of plate heat exchangers such as semi-welded PHE, fully welded PHE, tubular PHE and so on. The improvement of gasket material has made PHEs to withstand high temperatures and pressures. Due to these advances, today PHEs are also used in chemical process plants, OTEC (Ocean Thermal Energy Conversion) plants for evaporation and condensation duty and in nuclear power plants for their secondary circuits. To plan control and safety measures in such critical areas, it is necessary that accurate thermal models of the heat exchanger along with reliable heat transfer data be available for plant simulation.

The literature available in the area of thermal simulation of plate heat exchangers is vast. This includes the numerical and analytical models. Initially the plate heat exchanger thermal and flow characteristics had been studied for steady state operation, assuming plug flow inside the channel and an equal flow distribution from port to channels [1–4]. On the other hand, number of studies made on the transient performance of the plate heat exchangers is very limited. McKnight and Worley [5] initially worked in the transient analysis of PHEs with the application of feed back control to high velocity flow. Zeleski and Tajszerki [6] presented the simulation of the dynamic performance for co-current plate heat exchangers. The transient analysis of a counter-current plate heat exchanger subjected to flow transient was presented by Khan et al. [7]. Lakshmanan and Potter [8] developed a 'cinematic model' to predict the dynamic behaviour of single pass heat exchanger numerically. An extensive analysis was presented on the dynamics of the single pass PHE by Das and Roetzel [9]. In this model, it was considered that the axial thermal dispersion in both fluids, characterised by a dispersive Peclet number, takes care of all deviations from plug flow (flow maldistribution and fluid backmixing). In reality, inside the channels not only pure backmixing (Fourier-type dispersion, parabolic model) occurs but also flow maldistribution (non-uniformity of flow velocity). Both effects inside the channels could be taken into account with the hyperbolic dispersion model. However, this model would require another dimensionless parameter, the Mach number formed with flow and propagation velocity of thermal disturbances making the analysis more complex. For the present investigation it is reasonable to apply the simpler parabolic model, which requires the *Pe* number alone as extra measured parameter. The phase lag effect was introduced to account for delay at the distribution port. In this work, the temperature transients for unit step and sinusoidal response for U-type and Z-type configurations were analysed. The above dispersion model was validated by conducting experiments by, Das et al. [10]. Das and Murugesan [11] extended the above work for multi-pass plate heat exchangers. It was a general method that can be utilised for prediction of $1-n$, $m-n$, or $n-n$ pass PHEs. However, subsequent studies by Roetzel and Na Ranong [12,13] and Sahoo and Roetzel [14], confirmed that it is more appropriate to use axial dispersion for fluid backmixing rather than flow maldistribution. Hence, there is a need to treat flow maldistribution separately and more accurately.

It is also worth mentioning that, so far the transient studies assumed that the flow is equally distributed to all the channels and as a consequence the heat transfer coefficient is equal in each channel. But in reality the flow is unequally distributed from the port. So the velocity varies from channel to channel and hence the heat transfer coefficient as well. The effect of unequal distribution of fluid from port to channel was presented in detail using the analytical technique by Bajura and Jones [15] and Bassiouny and Martin [16,17]. It also brought out the difference between U-type and Z-type configurations of PHEs. This model is an extension of the manifold flow channelling theory of Bajura and Jones [15]. Datta and Majumdar [18] analysed the effect of unequal distribution of fluid inside the channel numerically. A numerical model presented by Thonon et al. [19] proved that pressure drop is significantly affected by maldistribution. Yang and Wang [20] presented the thermal performance and maldistribution in multi-pass PHEs. Similarly, Huang [21] developed a model about the port pressure and the channel flow rate distributions for single pass PHE. A steady state analysis was presented by Prabhakara Rao et al. [22,23] on the effect of flow distribution to the channels on the thermal performance of a PHE. In this study, it was considered that the heat transfer coefficient inside the channel is a function of the velocity of the fluid stream in that particular channel.

It is clear from the above survey of literature that analytical works on the steady state analysis of PHEs have been carried out extensively. The transient response of PHEs has also been done, but a uniform flow distribution has been assumed in these studies. The effect of port to channel maldistribution on dynamics has not been studied comprehensively. The previous transient studies assumed that maldistribution along with fluid back mixing can be taken care by using an axial dispersion within the channels, which is physically not very sound. So, the most realistic way is to calculate the flow maldistribution from the port to channel separately and the axial dispersion can be used to take care of fluid back mixing within the channel alone. This is the main inspiration of the present work.

The present analysis gives an improved form of analytical solution for the transient response of a single pass PHE. The maldistribution from channel to channel as well as the axial dispersion and the phase lag effect are considered. In this work, flow maldistribution is considered as suggested by Bajura and Jones [15] and Bassiouny and Martin [16,17]. So the velocity varies from channel to channel and hence the heat transfer coefficients are considered to be varying from channel to channel in this analysis. Accordingly the governing differential equations have been derived and solution has been obtained by taking proper boundary conditions into consideration. The results are presented for different values of maldistribution parameters as well as conventional heat exchanger parameters on temperature transients for single-pass PHEs. In addition to the simulated temperature responses the parameters of equivalent first order systems are also used to explain the results more comprehensively.

2. Mathematical formulation

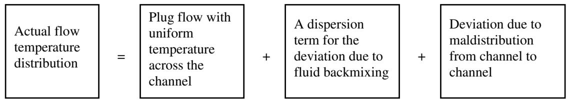
The single pass PHE is modelled mathematically, by considering the following assumptions. Some of the assumptions are same as those made for the transient analysis of the PHE with dispersion in both fluids in [9,11]. The main difference arises from the unequal distribution of the fluid from port to channel due to this the flow velocity and as a result the heat transfer coefficients are not identical for the channels carrying similar fluids.

Assumptions

1. All flow and thermal properties of the fluids are considered to be independent of temperature.
2. Heat transfer takes place only across the plates and not through the seals and gaskets.
3. The heat exchanger is thermally insulated from heat leak to the surroundings.
4. The fluid backmixing in the flow passages is taken care of by the Fourier-type axial dispersion term.
5. The flow maldistribution from channel to channel has been taken into account.
6. Heat transfer coefficient in each channel is considered to be a function of fluid velocity in that particular channel.
7. The plates are considered to be thin enough to neglect any axial conduction.
8. The heat exchanger is started from a uniform temperature for both the fluids.

Based on the literature it can be ascertained that the actual fluid flow in PHE deviates from the one-dimensional plug flow. All the deviations from plug flow are mainly due to the fluid back mixing in the channel, flow maldistribution, heat leakage through gaskets and seals etc. Here the maldistribution effects are separated from the axial dispersion, because the deviation due to maldistribution of the fluid from channel to channel has got more significant effect compared to its effect within the channel. So, instead of representing the axial dispersion for the deviations due to the sum of flow maldistribution and backmixing, it can take care of the deviations due to fluid back mixing inside the channel alone more effectively and flow maldistribution effects can be accounted for separately. While formulating the governing equations care has been taken in this analysis to consider the effects due to phase lag at channel entry.

We can now treat the flow within the channel as given below



The coordinate system is chosen in the direction of flow through the first channel. The numbers $1, 2, 3, \ldots, N$ (which is chosen to be odd) represents the channels, where an odd and even number of channels will carry fluid 1 and fluid 2, respectively. Because the total number of channels chosen is an odd number, cold fluid is allowed to flow through the odd channels such that end channels will have cold fluid to minimise the heat loss to the surroundings. In plate heat exchanger each plate is in contact with both hot and cold fluid on either side excepting the two end plates, which are in contact with only one fluid, and the other side is assumed to be insulated.

Small elements of fluid and plate can be considered as control volumes as shown in Fig. 1 to frame the energy balance equations for the fluid 1, fluid 2 and for all the plates considering the above assumptions. These equations are

Fluid 1:

$$\frac{C_1}{L}\frac{\partial\theta_i}{\partial\tau} = A_c D_i \frac{\partial^2\theta_i}{\partial X^2} - (-1)^{i-1}\dot{w}_i\frac{\partial\theta_i}{\partial X} + \frac{(hA)_i}{2L}(\theta_{Wi} + \theta_{Wi+1} - 2\theta_i) \quad \left(i = 1, 3, 5, \ldots, 2\left[\frac{N+1}{2}\right] - 1\right) \tag{1}$$

Fluid 2:

$$\frac{C_2}{L}\frac{\partial\theta_i}{\partial\tau} = A_c D_i \frac{\partial^2\theta_i}{\partial X^2} - (-1)^{i-1}\dot{w}_i\frac{\partial\theta_i}{\partial X} + \frac{(hA)_i}{2L}(\theta_{Wi} + \theta_{Wi+1} - 2\theta_i) \quad \left(i = 2, 4, 6, \ldots, 2\left[\frac{N}{2}\right]\right) \tag{2}$$

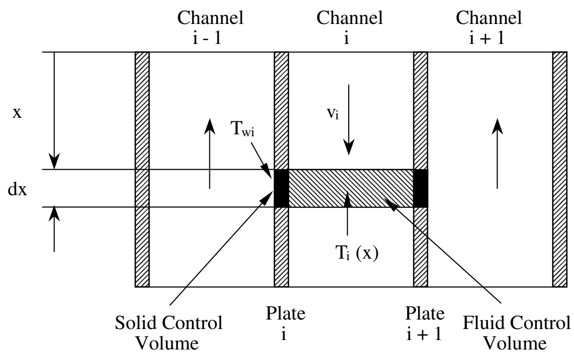


Fig. 1. Control volume of the fluid inside the channel and control volume of the plate.

For plates 2 to N:

$$\frac{C_W}{L}\frac{\partial\theta_{Wi}}{\partial\tau}=\frac{(hA)_{i-1}}{2L}(\theta_{i-1}-\theta_{Wi})+\frac{(hA)_i}{2L}(\theta_i-\theta_{Wi})\quad (i=2,3,4,\ldots,N) \tag{3}$$

For the plate 1:

$$\frac{C_W}{L}\frac{\partial\theta_{W1}}{\partial\tau}=\frac{(hA)_1}{2L}(\theta_1-\theta_{W1}) \tag{4}$$

For the plate $N+1$:

$$\frac{C_W}{L}\frac{\partial\theta_{W,N+1}}{\partial\tau}=\frac{(hA)_N}{2L}(\theta_N-\theta_{W,N+1}) \tag{5}$$

Number of variables involved in these equations is large, and can be reduced to a few non-dimensional terms, which are conventionally used to represent the heat exchanger parameters. Some of them are very important such as number of transfer units NTU, heat capacity rate ratio R_2 and dispersive Peclet number Pe, characterizing heat transfer, thermal balance and axial dispersion in fluids, respectively. Few more, like temperature, time and spatial coordinates can also be reduced to non-dimensional form. To consider the channel velocity variation from the uniform flow in all channels, a comparison is made with the uniform distribution. Based on this, remaining parameters also scaled accordingly. In case of heat transfer coefficient variation, it is varying with the velocity raised to the power 'n' from conventional heat transfer equation $Nu = C\cdot Re^n\cdot Pr^r$. In the present analysis the following set of dimensionless variables are chosen.

$$\tau_{\mathrm{ra1}}=\frac{C_1}{\dot{w}_{\mathrm{a1}}}\quad \tau_{\mathrm{ra2}}=\frac{C_2}{\dot{w}_{\mathrm{a2}}}$$

$$R_\tau=\frac{\tau_{\mathrm{ra2}}}{\tau_{\mathrm{ra1}}}\quad R_2=\frac{\dot{w}_{\mathrm{a2}}}{\dot{w}_{\mathrm{a1}}}\quad R_C=\frac{C_W}{C_1}$$

$$\mathrm{NTU}_i=U_{\mathrm{a1}}\cdot rh(i)\cdot rv(i)$$

$$U_{\mathrm{a1}}=\frac{hA}{\dot{w}_{\mathrm{a1}}}rv(i)=\frac{v_{\mathrm{a}}}{v_i}rh(i)=\frac{h_i}{h_{\mathrm{a}}}=\left[\frac{v_i}{v_{\mathrm{a}}}\right]^n$$

$$R_N=\frac{U_{\mathrm{a2}}r}{U_{\mathrm{a1}}}\quad R_{Wi}=\frac{\dot{w}_i}{\dot{w}_{\mathrm{a}}}\quad Pe_i=\frac{\dot{w}_iL}{A_{\mathrm{c}}D_i}$$

$$x=\frac{X}{L},\quad z=\frac{\tau}{\tau_{\mathrm{ra1}}},\quad t=\frac{\theta-\theta_{\mathrm{g1,in}}}{\theta_{\mathrm{g2,in}}-\theta_{\mathrm{g1,in}}}$$

with these dimensionless parameters, Eqs. (1)–(5) can be recast in the following non-dimensional form.

For both the fluids:

$$\frac{(R_\tau)^{m_{i+1}}}{R_{Wi}}\frac{\partial t_i}{\partial z}=\frac{1}{Pe_i}\frac{\partial^2 t_i}{\partial x^2}-(-1)^{i-1}\frac{\partial t_i}{\partial x}+\frac{\mathrm{NTU}_i}{2}(R_N)^{m_{i+1}}(t_{Wi}+t_{Wi+1}-2t_i)\quad (i=1,2,3,4,\ldots,N) \tag{6}$$

For intermediate plates except the end plates:

$$R_C\frac{\partial t_{Wi}}{\partial z}=K_{i-1}(t_{i-1}-t_{Wi})+K_i(t_i-t_{Wi})\quad (i=2,3,4,\ldots,N) \tag{7}$$

For the end plates:

$$R_C\frac{\partial t_{w1}}{\partial z}=\frac{\mathrm{NTU}_1}{2}R_{w1}(t_1-t_{w1}) \tag{8}$$

$$R_C\frac{\partial t_{w,N+1}}{\partial z}=K_N(t_N-t_{w,N+1}) \tag{9}$$

where $K_i=\frac{\mathrm{NTU}_i}{2}R_{Wi}(R_2\cdot R_N)^{m_{i+1}}$ and $m_j = j - 2[j/2]$.

Various ratios used in these equations are important and their values are related to the fluid streams within the channels and not the total values of the combined fluids flowing through the circuit. They are related to the values of ratios of the combined total stream as

$$R_2=R_{\mathrm{g2}}\left(\frac{n_1}{n_2}\right) \tag{10}$$

$$R_\tau=R_{\mathrm{g}\tau}\left(\frac{n_2}{n_1}\right) \tag{11}$$

3. Maldistribution parameter and non-dimensional volume flow rate

The most significant contribution in the present analysis is the inclusion of proper distribution of fluid in the channels from the port. As per Bajura and Jones [15] and Bassiouny and Martin [16,17] for the flow channelling of U-type configuration, the volumetric flow rate decreases along the flow direction in the entrance port and in case of Z-configuration, volumetric flow rate increases with the flow direction as per the correlations given in Table 1.

$$\dot{v}_{\mathrm{c}}=\frac{\text{Volume flow rate in the channel}}{\text{Mean volume flow rate for uniform flow}} \tag{12}$$

where distribution parameter 'm', given in the expressions mainly depends on the exchanger geometry, configuration and number of channels. In practical situations size of inlet and exit conduits are the same, in that case the parameter 'm^2' is proportional to the square of the ratio $(n\cdot A_c/A_p)$ and inversely proportional to the friction coefficient ζ_c of the channel flow.

$$m^2=\left(\frac{1}{\zeta_{\mathrm{c}}}\right)\cdot\left(\frac{n.A_{\mathrm{c}}}{A_p}\right)^2 \tag{13}$$

Since the friction coefficient depends on the chevron angle and Reynolds number, m^2 would not be exactly constant for each parallel channel, as assumed in this analysis. However an average value of 'm^2' matches closely with experiment as shown by Prabakara Rao and Das [24]. More over its value will increase with the

Table 1
The expressions for non-dimensional volume flow rate as a function of 'm' and flow arrangement

Configuration and distribution	m^2	w, $w^*\left(\frac{A^*}{A}\right)$	$\dot{v}_c$
For U-type and non-uniform	Positive	$\frac{\sinh m(1-y)}{\sinh m}$	$m\frac{\cosh m\cdot(1-y)}{\sinh m}$
For Z-type and non-uniform	Negative	$\frac{\sinh m\cdot y}{\sin hm}$	$m\frac{\cosh\cdot y}{\sinh m}$
For both types and uniform flow	0	$1-y$	1

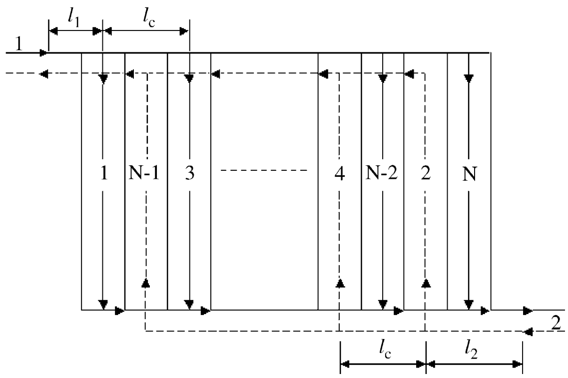


Fig. 3. Z-type configuration of plate heat exchanger.

square of the number of channels. Due to this reason, in heat exchangers having large number of channels, as the fluid flow in the conduit, few channels near the end will not carry any fluid, causing severe reduction in the performance of the heat exchanger. This is why maldistribution is having significant influence on the thermal performance of plate heat exchangers.

4. Phase lag and maldistribution effect

Plate heat exchangers show a special feature when compared to shell and tube heat exchanger namely phase lag effect. The fluid enters through the inlet port where the fluid is distributed eventually and enters into the channels $1, 2, 3, \ldots$ Hence the time at which the fluid stream enters the subsequent channels $1, 2, 3, \ldots$ has got increasing amount of phase lag and last channel will have maximum phase lag from the starting time when the combined fluid enters the heat exchanger at point 1, as shown in Fig. 2. In the present study, phase lag effect includes the influence of flow maldistribution. In case of U-type plate heat exchangers, more fluid will enter into the channel nearer to the inlet, fluid is distributed among all the channels such that the channel flow rate gradually decreases along the flow direction and the end channels will have minimum flow rates. Due this unequal distribution, velocity of the each channel varies, and this will also affect the phase lag giving an unequal rise in the flow direction inside the port, the equations for which are derived as given below. In case of Z-type configuration (Fig. 3) the flow maldistribution and phase lag effects are different, as the fluid flows in the inlet conduit the flow streams are distributed in a manner opposite to that in the U-type, such that the minimum flow rate is found in the first channel and gradually flow rate increases up to the last channel such that each fluid steam passes through the equal length of path within the heat exchanger. Because of the effect of malditribution on the phase lag, equations become more complex.

The velocity of the fluid inside the port decreases in the flow direction as the fluid streams leaving each channel (due to decrease in volume flow rate) as shown in Fig. 4. The velocities $V_1, V_2, V_3, \ldots$ are represented by velocities in the port after channels $1, 2, 3, \ldots,$ respectively. Before calculating the phase lag, it is required to find these velocities, for that we can apply the continuity condition at the entry of the channels and the ratios of these velocities can be obtained using the

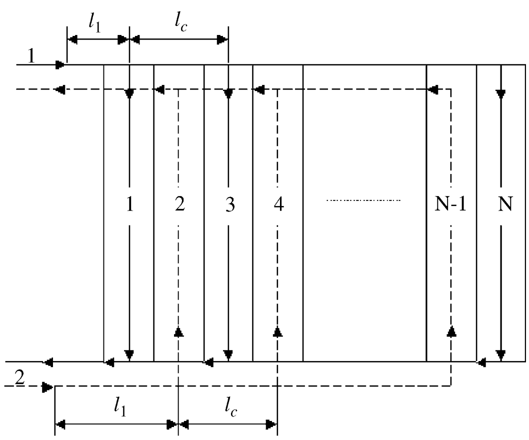


Fig. 2. U-type configuration of plate heat exchanger.

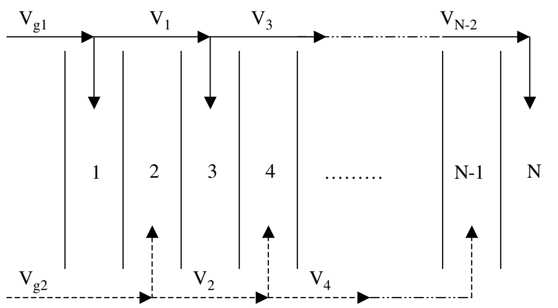


Fig. 4. Varying velocity in the inlet and exit ports.

non-dimensional volume flow rate in channels as given by Bajura and Jones [15] and Bassiouny and Martin [16,17].

The non-dimensional volume flow rate equation for the first channel can be written as

$$\dot{v}_{c1} = n1.\frac{\dot{V}_{c1}}{\dot{V}_{g1}} \tag{14}$$

Substituting Eq. (14) in to the continuity equation for the first channel gives

$$\frac{V_1}{V_{g1}} = 1 - \frac{\dot{V}_{c1}}{\dot{V}_{g1}} \tag{15}$$

Similarly, equation of the velocity ratio for the ith channel for both the fluids

$$\frac{V_{2i-1}}{V_{g1}} = 1 - \frac{1}{n1}\left[\sum_{j=1}^{i} \dot{v}_{c,2j-1}\right] \quad (i = 1, 2, 3, \ldots, n1) \quad \text{for fluid 1} \tag{16}$$

$$\frac{V_{2i}}{V_{g2}} = 1 - \frac{1}{n2}\left[\sum_{j=1}^{i} \dot{v}_{c,2j}\right] \quad (i = 1, 2, 3, \ldots, n2) \quad \text{for fluid 2} \tag{17}$$

The time required for the fluid to travel the distance between the channels.

('l_c' as shown in Figs. 2 and 3) can be calculated as

$$\Delta\tau_1 = l_1/V_{g1} \tag{18}$$

$$\Delta\tau_2 = l_2/V_{g2} \tag{19}$$

$$\Delta\tau_{2i+1} = \frac{l_c}{V_{2i-1}} \quad (i = 1, 2, 3, \ldots, n1 - 1) \tag{20}$$

$$\Delta\tau_{2i+2} = \frac{l_c}{V_{2i}} \quad (i = 1, 2, 3, \ldots, n2 - 1) \tag{21}$$

Hence the dimensionless phase lag between the entrances of consecutive channels may be expressed as

$$\Delta\phi_i = \Delta\tau_i/\tau_{ra1} \quad (i = 1, 2, 3, \ldots, N) \tag{22}$$

The total phase lag at the entry of each channel is the cumulated sum of the phase lags given by

$$\phi_{2i-1} = \sum_{j=1}^{2i-1} \Delta\phi_{2j-1} \quad (i = 1, 2, 3, \ldots, n1) \tag{23}$$

$$\phi_{2i} = \sum_{j=1}^{2i} \Delta\phi_{2j} \quad (i = 1, 2, 3, \ldots, n2) \tag{24}$$

After the fluid streams leaving from each channel inside the exit port, the phase lag encountered to reach the exit point can be computed in a similar way. Under the condition of dimensional symmetry in construction, as shown in Figs. 2 and 3, the relationship for this phase lag at exit can be reduced to

$$\phi_{i,\text{exit}} = \phi_i \quad \text{for a U-type plate exchanger} \tag{25}$$

and

$$\phi_{i,\text{exit}} = \phi_{n+1-i} \quad \text{for a Z-type plat exchanger.} \tag{26}$$

5. Boundary conditions

While estimating the boundary conditions attention is to be focussed at the entry of the channel. According to Danckwerts [25] if dispersion of the fluid begins at the entrance, a sudden temperature drop will be experienced at that section. The dispersion of the fluid inside the port is not taken into consideration because it is less significant. With this assumption, the boundary conditions for Eqs. (6)–(9) may be written as [25]

at $x = 0$:

$$t_i - \frac{1}{Pe_i}\frac{\partial t_i}{\partial x} = f_1(z - \phi_i)\cdot u(z - \phi_i) \quad \left(i = 1, 3, 5, \ldots, 2\left[\frac{N+1}{2}\right] - 1\right) \tag{27}$$

$$\frac{\partial t_i}{\partial x} = 0 \quad \left(i = 2, 4, 6, \ldots, 2\left[\frac{N}{2}\right]\right) \tag{28}$$

$$\frac{\partial t_{Wi}}{\partial x} = 0 \quad (i = 1, 2, 3, \ldots, N + 1) \tag{29}$$

at $x = 1$:

$$t_i - \frac{1}{Pe_i}\frac{\partial t_i}{\partial x} = f_2(z - \phi_i)\cdot u(z - \phi_i) \quad \left(i = 2, 4, 6, \ldots, 2\left[\frac{N}{2}\right]\right) \tag{30}$$

$$\frac{\partial t_i}{\partial x} = 0 \quad \left(i = 1, 3, 5, \ldots, 2\left[\frac{N+1}{2}\right] - 1\right) \tag{31}$$

$$\frac{\partial t_{Wi}}{\partial x} = 0 \quad (i = 1, 2, 3, \ldots, N + 1) \tag{32}$$

6. Solution for temperature distribution

The compete mathematical model is presented in the current analysis by using the $2N + 1$ partial differential equations expressed by Eqs. (6)–(9), along with the boundary conditions, represented by Eqs. (27)–(32). The initial condition for both the fluids and the walls may be taken to be uniform, since only starting from uniform temperature (cold state) is considered. Thus $t_{i,0} = t_{Wi,0} = 0$.

Taking the Laplace transform with respect to the reduced time variable z, we can solve this system of differential equations. With this operation, the system of Eqs. (6)–(9) is transformed into

$$\frac{d^2T_i}{dx^2} = Pe_i\left[\frac{R\tau^{m_{i+1}}}{R_{Wi}}\cdot s + \mathrm{NTU}_i(R_N)^{m_{i+1}}\right]T_i + (-1)^{i-1}Pe_i\frac{dT_i}{dx} - Pe_i\frac{\mathrm{NTU}_i}{2}(R_N)^{m_{i-1}}(T_{Wi}+T_{Wi+1})$$
$$(i = 1,2,3,\ldots,N) \quad (33)$$

From the plate equations one has

$$T_{Wi} = \frac{K_{i-1}}{M_{i-1}}\cdot T_{i-1} + \frac{K_i}{M_i}\cdot T_i \quad (34)$$

where $M_i = R_c \cdot s + K_{i-1} + K_i$.

By substituting Eq. (34) in Eq. (33), it reduces to the final single equation

$$\frac{d^2T_i}{dx^2} = (-1)^{i-1}\cdot Pe_i\frac{dT_i}{dx} - Pe_i(R_N)^{m_{i+1}}\frac{\mathrm{NTU}_i}{2}\frac{K_{i-1}}{M_i}\cdot T_{i-1} + Pe_i\left[\frac{R_\tau^{mi+1}}{R_{Wi}}\cdot s + (R_N)^{m_{i+1}}\frac{\mathrm{NTU}_i}{2}\left[2-\frac{K_i}{M_i}-\frac{K_i}{M_{i+1}}\right]\right]T_i - Pe_i(R_N)^{m_{i+1}}\frac{\mathrm{NTU}_i}{2}\frac{K_{i+1}}{M_{i+1}}\cdot T_{i+1}$$
$$(i = 1,2,3,\ldots,N) \quad (35)$$

Similarly the boundary conditions (27)–(32) can also be transformed into at $x = 0$

$$T_i - \frac{1}{Pe_i}\frac{dT_i}{dx} = F_1(s)\cdot e^{-\phi_i\cdot s} \quad \left(i = 1,3,5,\ldots,2\left[\frac{N+1}{2}\right]-1\right) \quad (36)$$

$$\frac{dT_i}{dx} = 0 \quad \left(i = 2,4,6,\ldots,2\left[\frac{N}{2}\right]\right) \quad (37)$$

$$\frac{dT_{Wi}}{dx} = 0 \quad (i = 1,2,3,\ldots,N+1) \quad (38)$$

at $x = 1$:

$$T_i - \frac{1}{Pe_i}\frac{dT_i}{dx} = F_2(s)\cdot e^{-\phi_i\cdot s} \quad \left(i = 2,4,6,\ldots,2\left[\frac{N}{2}\right]\right) \quad (39)$$

$$\frac{dT_i}{dx} = 0 \quad \left(i = 1,3,5,\ldots,2\left[\frac{N+1}{2}\right]-1\right) \quad (40)$$

$$\frac{dT_{Wi}}{dx} = 0 \quad (i = 1,2,3,\ldots,N+1) \quad (41)$$

The transformed Eq. (35) can be expressed, in the matrix form as

$$\frac{d\overline{T}}{dx} = \overline{A}\overline{T} \quad (42)$$

where the temperature vector $\overline{T}$ is given by

$$\overline{T} = \left[T_1, T_2, \ldots, T_N, \frac{dT_1}{dx}, \frac{dT_2}{dx}, \ldots, \frac{dT_N}{dx}\right]^{\mathrm{T}} \quad (43)$$

and the coefficient matrix $\overline{A}$ can be written as

$$A_{i,N+i} = 1 \quad (i = 1,2,3,\ldots,N)$$

$$A_{N+1,1} = Pe_1\left[\frac{1}{R_{W1}}\cdot s + \frac{\mathrm{NTU}_1}{2}\left[2-\frac{K_1}{M_1}-\frac{K_1}{M_2}\right]\right]$$

$$A_{N+1,2} = -Pe_1R_N\frac{\mathrm{NTU}_1}{2}\frac{K_2}{M_2}$$

$$A_{2N,N-1} = -Pe_N(R_N)^{m_{i+1}}\frac{\mathrm{NTU}_N}{2}\frac{K_{N-1}}{M_N}$$

$$A_{2N,N} = Pe_N\left[\frac{1}{R_{WN}}\cdot s + (R_N)^{m_{i+1}}\frac{\mathrm{NTU}_N}{2}\left[2-\frac{K_N}{M_N}-\frac{K_N}{M_{N+1}}\right]\right]$$

$$A_{N+i,i-1} = -Pe_i(R_N)^{m_{i+1}}\frac{\mathrm{NTU}_i}{2}\frac{K_{i-1}}{M_i}$$

$$A_{N+i,i} = Pe_i\left[\frac{R\tau^{m_{i+1}}}{R_{Wi}}\cdot s + (R_N)^{m_{i+1}}\frac{\mathrm{NTU}_i}{2}\left[2-\frac{K_i}{M_i}-\frac{K_i}{M_{i+1}}\right]\right]$$

$$A_{N+i,i+1} = -Pe_i(R_N)^{m_{i+1}}\frac{\mathrm{NTU}_i}{2}\frac{K_{i+1}}{M_{i+1}}$$

$$A_{N+i,N+i} = -(-1)^{i+1}\cdot Pe_i. \quad (i = 1,2,3,\ldots,N)$$

All elements of matrix $\overline{A}$ other than those described above are zero.

By deriving the eigenvalues β_j and eigenvectors (u_j) of the coefficient matrix $\overline{A}$ the solution to Eq. (42) can be obtained. This is a simple boundary value problem with distance coordinate as the only variable. The solution is

$$\overline{T} = \overline{U}\cdot\overline{B}(x)\cdot\overline{D} \quad (44)$$

where $\overline{B}(x)$ is a diagonal matrix.

$$\overline{B}(x) = \mathrm{diag}\{e^{\beta_1\cdot x}, e^{\beta_2\cdot x}, \ldots, e^{\beta_{2N}\cdot x}\} \quad (45)$$

The columns of the matrix $\overline{U}$ are corresponding eigenvectors of $\overline{A}$ and $\overline{D}$ is a coefficient vector, which depends on the boundary conditions, given by Eqs. (36)–(41). The fluid temperature distribution and its derivative can thus be expressed as

$$T_i = \sum_{j=1}^{2N} d_j\cdot u_{ij}e^{\beta_j\cdot x} \quad (i = 1,2,3,\ldots,N) \quad (46)$$

$$\frac{dT_i}{dx} = \sum_{j=1}^{2N} d_j\cdot u_{n+i,j}e^{\beta_j\cdot x} \quad (i = 1,2,3,\ldots,N) \quad (47)$$

The coefficient matrix $\overline{D}$ can be determined by substituting Eqs. (46) and (47) into the boundary conditions (36)–(41), to obtain the matrix equation

$$\overline{W}\cdot\overline{D} = \overline{S} \quad (48)$$

The right hand vector $\overline{S}$ contains the inlet temperature functions along with its phase lag and can be written as

$$\overline{S} = [F_1(S)\mathrm{e}^{-\phi_1 \cdot s}, F_2(S)\mathrm{e}^{-\phi_2 \cdot s}, F_1(S)\mathrm{e}^{-\phi_3 \cdot s}, F_2(S)\mathrm{e}^{-\phi_4 \cdot s}, \ldots, F_k(S)\mathrm{e}^{-\phi_N \cdot s}, 0, 0, 0, \ldots, 0] \tag{49}$$

where $k = 1$ for odd N and $k = 2$ for even N.

Hence $\overline{D}$ can be obtained from

$$\overline{D} = \overline{W}^{-1}\overline{S} \tag{50}$$

7. Response in time domain

Temperature response in the frequency domain can be obtained by solving Eq. (42), which has to be inverted into the time domain by Laplace inversion. The expressions are too complex to carry out the inversion analytically. Hence, the Laplace inversion by Crump's [26] numerical algorithm using Fourier series approximation is used in the present analysis. Any function $g(Z)$ with a Laplace transform $G(s)$ can be expressed as

$$g(Z) = \frac{\exp(aZ)}{Z}\left[\frac{1}{2}G(a) + Re\sum_{k=i}^{\infty} G\left(a + \frac{ik\pi}{Z}\right)(-1)^k\right] \tag{51}$$

The constant a is chosen in the domain $4 < aZ < 5$ to minimize the truncation error. It can be further simplified by using Fast Fourier Transform. Substituting $Z = 2nZ/M$, the above equation results in

$$g(Zn) = \frac{\exp(aZ_n)}{Z}\left[Re\sum_{k=1}^{M-1} G\left(a + \frac{ik\pi}{Z}\right) \cdot \exp\left(i.\frac{2\pi nk}{M}\right) - \frac{1}{2}g(a)\right] \tag{52}$$

The term indicated by summation is obtained by Fast Fourier Transformation at every point Z_n in that domain. The final temperature of the combined fluid at the outlet can be calculated by considering temperature responses at the exit of the each channel and corresponding phase lag, and using the weighted mean average as follows

$$t(z) = \frac{\sum \dot{m}_i \cdot t_i(z - \phi_i)}{\sum \dot{m}_i} \tag{53}$$

8. Results and discussion

Temperature transients can be calculated due to change in either of the fluid temperatures (or both) by using the above-mentioned procedure. The effects of flow maldistribution parameter m^2, dispersion parameter Pe along with conventional heat exchanger parameters such as NTU, heat capacity rate ratio R, and number of channels N, on the transients are presented here. The results are obtained based on the realistic values of the heat exchanger parameters and geometrical configurations. The distance between the two consecutive plates is assumed to be 4% of the effective length of the plates, and also assumed that port sizes for inlet and exit conduits are equal where flow bifurcation and mixing occurs. The entry temperature of fluid 2 is subjected to rise as a unit step with time.

The aim of the present work is to develop a general model for transient simulation considering effect of flow maldistribution, instead of bringing out the temperature responses for different types of inputs. Hence, only response to step inputs has been calculated for different combinations of parameters along with different degrees of maldistribution. To obtain a clear understanding about the different transients, the response is ideally characterised by second order system response with delay time. However, the second order response incorporates too many constants, the physical significance of which are not always obvious for over damped systems. On the other hand the first order response approximation is not very inaccurate, except for the mismatch of the slope during the first few percent of temperature rise. Hence it is suggested that the delay time and time constant based on first order system be utilised for the control of the heat exchangers. So the same transient response results can be well described by comparing with the first order system characteristics. Such a comparison helps to suggest an approximate transfer function, which can be controlled by commonly used control loops

$$G = \frac{K\mathrm{e}^{-\tau_d \cdot s}}{1 + \tau_c \cdot s} \tag{54}$$

where τ_c is the time constant, τ_d the delay time and K the gain.

The values of the equivalent first order system parameters have been determined using non-linear regression analysis of the computed data for the temperature responses.

8.1. Effect of flow maldistribution and heat exchanger parameters

The values considered for this parametric study are $N = 25$, $R_c = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $NTU_1 = 1.0$ and $Pe = 20$. Variation of Peclet number with the Reynolds number is not much significant within the small range [27], so the variation of Peclet number for different channels is not taken into account and the same value is taken for the both the fluids. For all the responses Peclet number of 20 is taken in the present study as against 5 taken in the previous studies [9,11]. This is because in the present analysis Peclet number indicates only the back mixing of the fluid inside the channel and maldistribution factor is considered separately, whereas in the previous studies [9,11], the Peclet number takes care of both the back mixing inside the channel as well as the flow maldistribution from port to channel.

It is clear that the dimensionless outlet temperature of the cold fluid at steady state represents the effectiveness of the heat exchanger since it is assumed that initially both the fluids are at the same temperature and heat capacity ratio is unity. Figs. 5 and 6 represent the temperature responses for both cold and hot side of U-type plate heat exchanger, respectively. It is observed that cold side outlet temperature decreases at steady state with maldistribution, which means the effectiveness of the heat exchanger reduces with maldistribution. It is evident that steady state temperatures of hot and cold fluid add up to unity, which satisfies the energy balance condition for equal heat capacity rates. For cold side, the initial delay in response is independent of maldistribution and initially the response time decreases with the increase of maldistribution and the trend is reversed as it approaches the steady state in conformity with the fact that heat exchanger effectiveness decreases with the increase of maldistribution. The faster response time is observed during initial period is due to the fact that the channels, which are nearer to the inlet, carry more fluid due to flow maldistribution.

For Z-type heat exchanger (Fig. 7), the outlet temperature of the cold fluid decreases at steady state with m^2, which means that effectiveness of the heat exchanger reduces with the increase of maldistribution as in U-type. It is also observed that the delay time is more in case of Z-type configuration and increases with the 'm^2' since the fluid has to travel across all the channels to give the temperature rise at the outlet. As a result crossover of the transient curves does not take place as observed in 'U' type exchanger.

The effect of maldistribution and Peclet number on the temperature transients for cold and hot sides is represented in Figs. 8 and 9. Here PHE with minimum and maximum Peclet numbers 4 and 30 is taken into consideration and it is observed that the cold side temperature increases with the dispersive Peclet number because of the axial dispersion inside the channel, i.e., the fluid

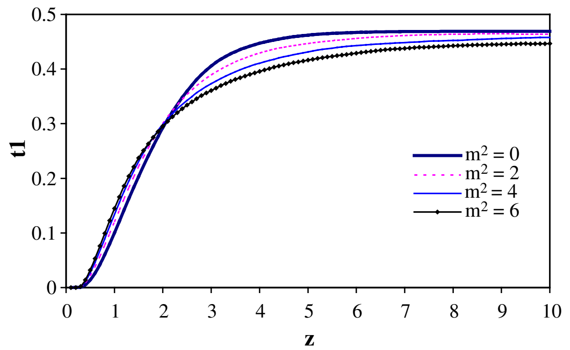


Fig. 5. For U-type: effect of maldistribution for $N = 25$, $Pe = 20$, NTU = 1, and $R = 1$ (cold).

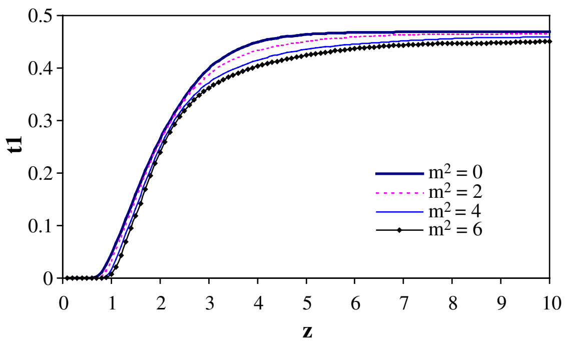


Fig. 7. For Z-type: effect of maldistribution for $N = 25$, $Pe = 20$, NTU = 1, and $R = 1$ (cold).

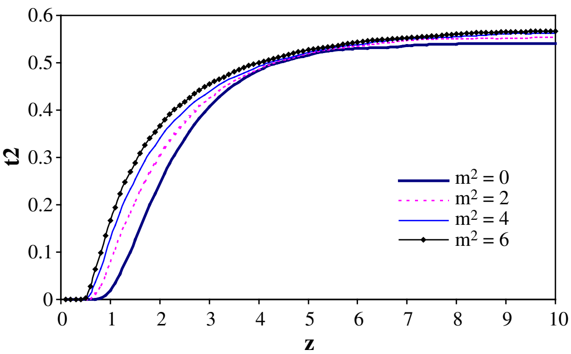


Fig. 6. For U-type: effect of maldistribution for $N = 25$, $Pe = 20$, NTU = 1, and $R = 1$ (hot).

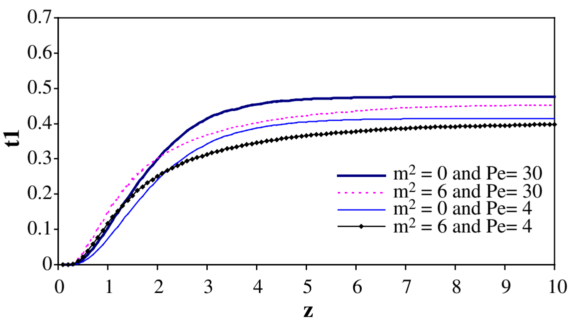


Fig. 8. Effect of maldistribution and Peclet number for $N = 25$, NTU = 1, and $R = 1$ (cold).

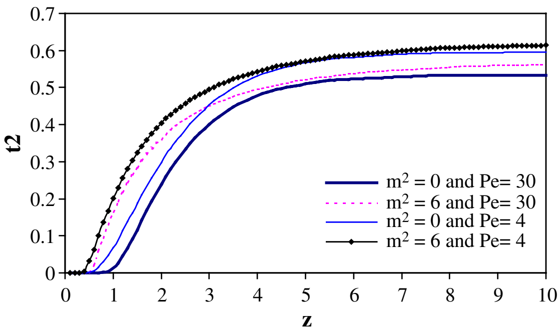


Fig. 9. Effect of maldistribution and Peclet number for $N = 25$, NTU = 1, and $R = 1$ (hot).

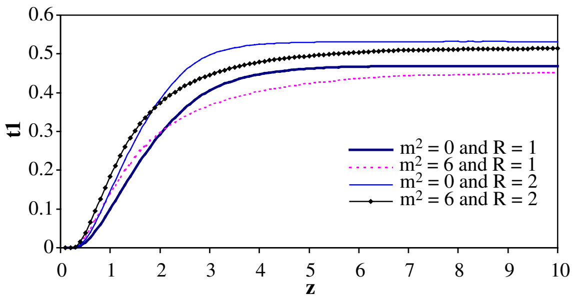


Fig. 11. Effect of maldistribution and NTU for $N = 25$, $Pe = 20$, and $R = 1$ (cold).

backmixing. At the same time the cold fluid temperature decreases with maldistribution and so also the effectiveness. With this it can be easily concluded that the performance of the heat exchanger will decrease not only with the maldistribution but also with dispersion in the fluid. It can also be observed that the effect of maldistribution is more predominant at low axial dispersion (higher *Pe*) than at high dispersion (low *Pe*). This is because, with strong axial dispersion the additional energy transfer through fictitious conduction compensates for the maldistribution effect to some extent. The hot side responses show just the opposite trends at the steady state, for obvious reasons. Similarly, the effects of the maldistribution and other parameters like NTU, heat capacity rate ratio and number of channels on temperature responses are on the expected lines. As NTU value increases from 1 to 5, cold side temperature (i.e., effectiveness) increases. The deterioration of the performance of the heat exchanger due to maldistribution is almost independent of the change in NTU (Fig. 10) as well as heat capacity rate ratio (Fig. 11). Moreover it is interesting to observe that for increasing number of channels, the performance reduction due to maldistribution decreases, and thermal performance gets enhanced due to the reduction in end effects as shown in Fig. 12. Since maldistribution parameter '*m*' itself increases with the number of channels, this result essentially indicates the effect of channel to port area as well as channel friction coefficient. Smaller the port area and the friction coefficient, larger is the extent of effect on heat exchanger performance at lower number of plates compared to higher number of plates. From the nature of the transient parts of the curves it can be ascertained that NTU and *R* does not influence the maldistribution effect significantly but number of channels has got a considerable influence on maldistribution effect.

It is difficult to distinguish the transients because of closeness of the responses. Hence the responses are approximated by first order response due to the reasons stated earlier. Here the gain in the first order system (FOS) represents the steady state temperature, so the cold side value of the temperature, i.e., effectiveness (for $R = 1$) of the heat exchanger is the gain in FOS. The other first order parameters, which are taken into consideration, are delay time, and time constant.

From the Figs. 13 and 14, it can be observed that the gain (effectiveness) reduces marginally with the maldistribution in both the U- and Z-type configurations. For U-type, reduction in delay time is due to the fitting problem related to the faster response time which is

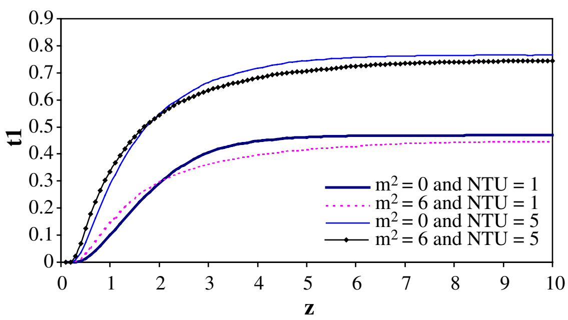


Fig. 10. Effect of maldistribution and heat capacity rate ratio for $N = 25$, NTU = 1, and $Pe = 20$ (cold).

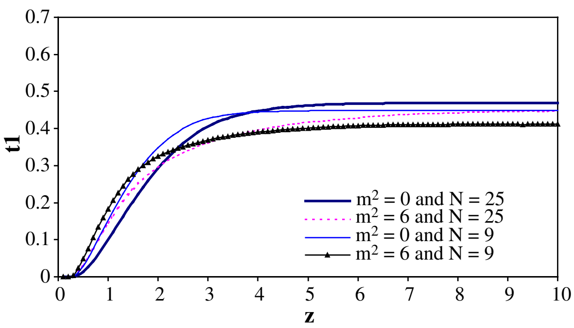


Fig. 12. Effect of maldistribution and number of channels for $Pe = 20$, NTU = 1, and $R = 1$ (cold).

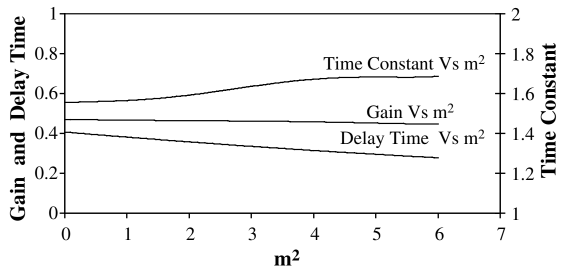


Fig. 13. For U-type: parameters of first order system against m^2.

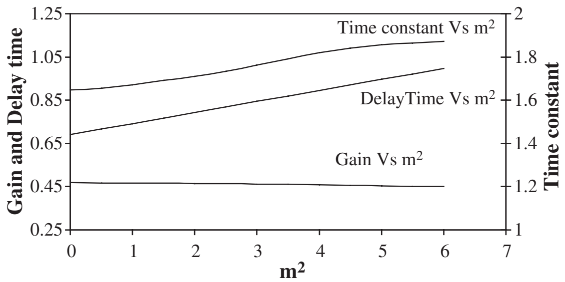


Fig. 14. For Z-type: parameters of first order system against m^2.

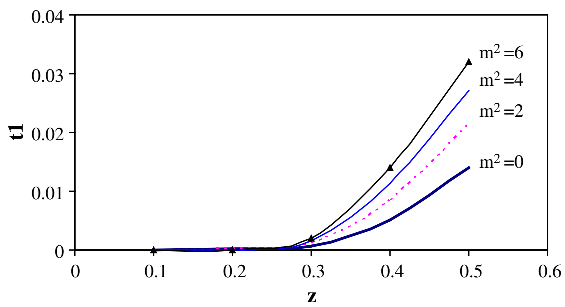


Fig. 15. Close view of effect of maldistribution for $N = 25$, $Pe = 20$, NTU = 1, and $R = 1$ (cold).

observed during initial period with increase in 'm^2'. This can be observed from the close view of the response as shown in Fig. 15. In case of Z-type, the delay time increases significantly in reality with maldistribution. The time constant increases with maldistribution slowly and reaches steady state in both U-type and Z-type.

9. Conclusion

A comprehensive strategy for analytical treatment of the transient behaviour of plate heat exchangers is presented. Emphasis is given on the effects of port to channel flow maldistribution on the transient thermal behaviour. The treatment of the flow maldistribution separately from axial dispersion which takes care of fluid backmixing is a distinctive feature of the present analysis. This allows quantifying the effect of flow maldistribution and its influence on the heat exchanger thermal behaviour. The unequal phase lag of the temperature transient at the entry of each channel arising out of unequal flow distribution has also been taken into account. A Laplace transform technique used to convert the system of PDEs to ODEs followed by matrix eigenvalue solution and subsequent numerical Laplace inversion using FFT has been the adopted technique here. The results clearly indicate that flow maldistribution plays a major role in the thermal performance of PHEs both in the steady and transient domain. The influence of maldistribution is sensitive to the axial dispersive Peclet number and number of plates while it is almost independent of NTU and heat capacity rate ratio. It is observed that transient signatures such as the initial delay, time constant and the asymptotic response critically depend on the extent of maldistribution for U-type and Z-type configuration. The present results are of practical importance not only for the proper design of the heat exchanger to minimise maldistribution but also to impart proper control strategy for the off normal behaviour of the heat exchanger.

Acknowledgement

The authors gratefully acknowledge the financial support from SIDA-Swedish research links and Swedish research council (VR) for the project under which the present collaborative research was carried out.

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